Rayleigh-Taylor Instability of a Stratified Plasma

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We have investigated the effect of finite Larmor radius on the Rayleigh-Taylor instability of a semi-infinite, compressible, stratified and infinitely conducting plasma. The plasma is assumed to have a one dimensional density and magnetic field gradients. The eigenvalue problem has been solved under Boussinesq approximation for disturbances parallel to the magnetic field. It has been established that for perturbation parallel to the magnetic field, the system is stable for both stable and unstable stratification. For perturbation perpendicular to the magnetic field, the problem has been solved without Boussinesq approximation. The dispersion relation has been discussed in the two limiting cases, the short and long wave disturbances. It has been observed that the gyroviscosity has a destabilizing influence from $k = 0$ to $k = 4.5$ for $\beta^* = 0.1$ and for $\beta^* = 0.1$ up to $k^* = 2.85$ and then onwards it acts as a stabilizing agent. It has a damping effect on the short wave disturbances. For some parameters, the largets imaginary part has been shown in Figs. 1 and 2.

1. Introduction

Chandrasekhar [1] has given a detailed account of the investigation of the Rayleigh-Taylor instability as carried out by various research workers under varying assumptions. Rosenbluth, Krall and Rostokar [2], Roberts and Taylor [3] and Jukes [4] have pointed out the stabilizing influence of F.L.R. effects on a plasma, which exhibits itself in the form of magnetic viscosity in the fluid equations.

Melchior and Popovich [5] have studied the effect of F.L.R. on the Kelvin-Helmholtz instability of a fully ionized plasma. Ariel and Bhatia have confirmed the stabilizing influence of F.L.R. on the Rayleigh-Taylor instability of a semi-infinite plasma [6] and rotating plasma [7]. Talwar [8] studied the Rayleigh-Taylor instability of a rotating fluid of variable density omitting F.L.R. effects. In this paper we have studied the F.L.R. effects on the Rayleigh-Taylor instability of a plasma of variable density and variable magnetic field. The semi-infinite plasma is assumed to have a one dimensional density and magnetic field gradients. The plasma is supposed to be compressible and infinitely conducting. The cases of longitudinal and transverse perturbations have been discussed separately. The study of the coupled modes is rather unwieldy.

2. Linearized Equations

We consider a stratified fluid contained between two planes $z = 0, a$ in equilibrium under the action of a variable horizontal magnetic field $H_0(z)$ along the x-axis and a gravitational field $\boldsymbol{g} = (0, 0, -g)$ pointing downwards. The fluid is assumed to be compressible and infinitely conducting. The linearized hydromagnetic equation are:

$$\frac{\partial}{\partial t} + \nabla \cdot \varrho_0 \, \boldsymbol{u}) = 0 , \tag{1}$$

$$\varrho_0 \frac{\partial \boldsymbol{u}}{\partial t} = - \nabla \cdot \overleftrightarrow{\delta \mathrm{P}} + \boldsymbol{g} \, \delta \varrho + \frac{\mu}{4\,\pi} \cdot$$
$$\cdot \ [(\nabla \times \boldsymbol{H}_0) \times \boldsymbol{h} + (\nabla \times \boldsymbol{h}) \times \boldsymbol{H}_0] , \tag{2}$$

$$\frac{\partial \delta p}{\partial t} + \boldsymbol{v} \cdot \nabla p_0 = \frac{r \, p_0}{\varrho_0} \left(\frac{\partial \delta \varrho}{\partial t} + \boldsymbol{u} \cdot \nabla \varrho_0 \right), \tag{3}$$

$$\partial \boldsymbol{h} / \partial t = \nabla \times (\boldsymbol{u} \times \boldsymbol{H}_0) , \tag{4}$$

where δp is the perturbation in the isotropic part of the pressure tensor and

$$\overleftrightarrow{\delta \mathrm{P}} = \overleftrightarrow{\delta p} + \overleftrightarrow{\delta P^*} \tag{5}$$

and $\boldsymbol{u}$, $\boldsymbol{h}$, $\delta \varrho$ and $\overleftrightarrow{\delta P^*}$ respectively denote the perturbations in velocity, magnetic field, density and the pressure tensor taking into account the F.L.R. effects.

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For the magnetic field acting along the x-axis $\overleftrightarrow{\delta P}$ is given by

$$\delta P_{xx} = \delta p \,,$$

$$\delta P_{yy} = \delta p - \varrho_0 \, \nu \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right)$$

$$\delta P_{zz} = \delta p + \varrho_0 \, \nu \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right)$$

$$\delta P_{xy} = \delta P_{yx} = - 2 \, \varrho_0 \, \nu \, \partial u / \partial z$$

$$\delta P_{xz} = \delta P_{zx} = 2 \, \varrho_0 \, \nu \, \partial u / \partial z$$

$$\delta P_{yz} = \delta P_{zy} = \varrho_0 \, \nu \left(\frac{\partial v}{\partial y} - \frac{\partial w}{\partial z} \right) \tag{6}$$

where

$$\varrho_0 \, \nu = N \, T / (4 \, \omega_c) \,.$$

ω_c being the ion-gyration frequency while N and T are respectively the number density and the temperature of the ions. The z-axis has been taken in the vertical upward direction. It is worthwhile to note that the gyro-viscosity, ν, is not uniform.

We introduce the displacement vector $\boldsymbol{\xi}$, such that

$$\boldsymbol{u} = \partial \boldsymbol{\xi} / \partial t \,, \tag{7}$$

where

$$\boldsymbol{\xi} = x_i \, \xi_x + y_i \, \xi_y + z_i \, \xi_z \,, \tag{8}$$

x_i, y_i, z_i being the unit vectors along the x, y, z axes respectively.

At this stage, we assume that the magnetic pressure is proportional to the mechanical pressure and that the medium is isothermal. The initial distributions are chosen so as to satisfy the equilibrium equation

$$\frac{\mathrm{d}}{\mathrm{d}z} \left(p_0 + \frac{\mu \, H_0{}^2}{8 \, \pi} \right) = - g \, \varrho_0 \,. \tag{9}$$

The distributions taken are as under

$$\varrho_0 = \varrho_s \exp(- \beta \, z) \,, \quad H_0 = H_s \exp(- \beta \, z / 2) \,, \tag{10}$$

where ϱ_s and H_s are the density and the magnetic field at $z = 0$.

Eliminating $\delta \varrho$, δp, $\overleftrightarrow{\delta P^*}$ and $\boldsymbol{h}$ from Eq. (2) with the help of Eqs. (1), (3), (4), (5) and (10) we obtain

$$\partial^2 \boldsymbol{\xi} / \partial t^2 = [g - (c^2 + V^2) \beta] \, (\nabla \cdot \boldsymbol{\xi}) z_i + (g - \tfrac{1}{2} V^2 \beta) \, \nabla \, (\boldsymbol{\xi} \cdot z_i) + c^2 \, \nabla \, (\nabla \cdot \boldsymbol{\xi})$$

$$+ V^2 \{ x_i \times [\nabla \, (\nabla \cdot \boldsymbol{\xi}) \times x_i - \nabla \times (x_i \cdot \nabla) \, \boldsymbol{\xi}] - [x_i \cdot (x_i \nabla) \, \boldsymbol{\xi}] z_i \} + \nu_1 \, (\partial / \partial t)$$

$$\cdot \left[\beta \, \frac{\partial \xi_x}{\partial y} \, x_i + y_i \left\{ 2 \, \frac{\partial^2 \xi_x}{\partial x \, \partial z} + \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \xi_z + \frac{\beta}{2} \left(\frac{\partial \xi_y}{\partial z} - \frac{\partial \xi_z}{\partial y} \right) \right\} \right.$$

$$\left. - z_i \left\{ 2 \, \frac{\partial^2 \xi_x}{\partial x \, \partial y} + \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \xi_y - \frac{\beta}{2} \left(\frac{\partial \xi_z}{\partial_y} + \frac{\partial \xi_y}{\partial z} \right) \right\} \right], \tag{11}$$

where c and V are the local sound and Alfvén speeds defined by

$$c^2 = \gamma \, p_0 / \varrho_0 \,, \quad V^2 = \mu \, H_0{}^2 / (4 \, \pi \, \varrho_0) \,,$$

$$\nu_1 = \frac{c \, T}{4 \, e \, H_s} \exp(\beta \, z / 2) \,. \tag{12}$$

Analysing in terms of normal modes we seek solutions whose dependence is of the form

$$\boldsymbol{\xi} = \boldsymbol{\xi} (z) \exp i \, (\boldsymbol{k} \cdot \boldsymbol{r} - \omega \, t) \,, \tag{13}$$

where k, ω and $\boldsymbol{\xi}$ denote respectively the wave number vector, the frequency and the amplitude of the wave. Equation (12) with the help of Eq. (13) breaks up into three component equations:

$$(\omega^2 - c^2 \, k_x{}^2) \, \xi_x - c^2 \, k_x \, k_y \, \xi_y - i \, k_x \, g \, \xi_z + c^2 \, i \, k_x \, D \, \xi_z + \nu_1 \, \beta \, \omega \, k_y \, \xi_x = 0 \,, \tag{14}$$

$$[2 \, \nu_1 \, \omega \, k_x \, D - c^2 \, k_x \, k_y] \xi_x + [\omega^2 - (c^2 + V^2) \, k_y{}^2 - V^2 \, k_x{}^2 + \nu_1 \, \omega \, \beta \, k_y] \xi_y$$

$$+ \, [(c^2 + V^2) \, i \, k_y \, D - i \, g \, k_y - i \, \omega \, \nu_1 \, (D^2 - k_x{}^2) + i \, \omega \, \nu_1 \, \beta \, D] \xi_z = 0 \,, \tag{15}$$

$$i \, k_x [(g - c^2 \, \beta) + c^2 \, D - 2 \, \nu_1 \, \omega \, k_y] \xi_x + i \, [(g - (c^2 + V^2) \, \beta) \, k_y$$

$$+ \, (c^2 + V^2) \, k_y \, D + \nu_1 \, \omega \, (D^2 - k_y{}^2) - \tfrac{1}{2} \, \nu_1 \, \omega \, \beta \, D] \xi_y + [\omega^2 + (c^2 + V^2) \, D^2$$

$$- \, (c^2 + V^2) \, \beta \, D - V^2 \, k_x{}^2 + \tfrac{1}{2} \, \nu_1 \, \omega \, \beta \, k_y] \xi_z = 0 \,. \tag{16}$$

The dimensionless forms of these equations are

$$\xi^*_x [\omega^{*2} - k^*_x{}^2 + \nu^*_1 \, \omega^* \, \beta^* \, k^*_y] - k^*_x \, k^*_y \, \xi^*_y + i \, k^*_x \, (D^* - g^{*2}) \, \xi^*_z = 0 \,, \tag{17}$$

$$k^*_x[2\,\nu^*_1\,\omega^*\,D^* - k^*_y]\xi^*_x + [\omega^{*2} - (1 - S^2)k^{*2}_y - S^2\,k^{*2}_x + \tfrac{1}{2}\,\nu^*_1\,\omega^*\,\beta^*\,k^*_y]\xi^*_y$$
$$+ i[(1 + S^2)k^*_y\,D^* - g^{*2}\,k^*_y - \nu^*_1\,\omega^*\,(D^{*2} - k^{*2}_x) + \tfrac{1}{2}\,\nu^*_1\,\omega^*\,\beta^*\,D^*]\xi^*_z = 0\,, \tag{18}$$

$$i\,k^*_x[g^{*2} - \beta^* + D^* - 2\,\nu^*_1\,\omega^*\,k^*_y]\xi^*_x + i[(1 + S^2)k^*_y\,D^* - \tfrac{1}{2}\,\nu^*_1\,\omega^*\,\beta^*\,D^*$$
$$+ k^*_y\{g^{*2} - (1 + S^2)\beta^*\} + \nu^*_1\,\omega^*\,(D^{*2} - k^{*2}_y)]\,\xi^*_y + [\omega^{*2} + (1 + S^2)D^{*2}$$
$$- (1 + S^2)\beta^*\,D^* - k^{*2}_x\,S^2 + \tfrac{1}{2}\,\nu^*_1\,\omega^*\,\beta^*\,k^*_y]\,\xi^*_z = 0\,, \tag{19}$$

where the non-dimensional numbers are

$$S^2 = V^2/c^2\,, \qquad g^{*2} = g\,a/c^2\,, \qquad \omega^* = \omega\,a/c\,, \qquad k^*_x = k_x\,a\,, \qquad k^*_y = k_y\,a\,, \qquad \beta^* = \beta\,a\,,$$
$$\xi^*_x = \xi_x/a\,, \qquad \xi^*_y = \xi_y/a\,, \qquad \xi^*_z = \xi_z/a\,, \qquad \nu_1 = \nu_1/a\,c\,, \qquad D^* = a\,D\,. \tag{20}$$

Following Hosking and Marinoff [9] for stability we write

$$\nabla \cdot \boldsymbol{\xi} = 0\,, \tag{21}$$

which is also known as the Boussinesq approximation. The analysis is carried out for longitudinal and transverse oscillations separately. For longitudinal perturbations we write $k^*_x = k^*$ and $k^*_y = 0$, then the Eqs. $(17 - 21)$ become

$$(\omega^{*2} - k^{*2})\xi^*_x - i\,k^*\,g^{*2}\,\xi^*_z + i\,k^*\,D^*\,\xi^*_z = 0\,, \tag{22}$$

$$2\,\nu^*_1\,\omega^*\,k^*\,D^*\,\xi^*_x + (\omega^{*2} - k^{*2}\,S^2)\xi^*_y - i\,\omega^*[\nu^*_1\,(D^{*2} - k^{*2}) - \tfrac{1}{2}\,\nu^*_1\,\beta^*]\,\xi^*_z = 0\,, \tag{23}$$

$$[i(g^{*2} - \beta^*)k^* + i\,k^*\,D^*]\xi^*_x + i\,\omega^*[\nu^*_1\,D^{*2} - \tfrac{1}{2}\,\nu^*_1\,\beta^*\,D^*]\xi^*_y$$
$$+ [\omega^{*2} + (1 + S^2)D^{*2} - k^{*2}\,S^2 - (1 + S^2)\beta^*\,D^*]\xi^*_z = 0\,, \tag{24}$$

$$i\,k^*\,\xi^*_x + D^*\,\xi^*_z = 0\,. \tag{25}$$

Equation (25) with the help of (28) gives

$$\omega^{*2}\,\xi^*_x - i\,k^*\,g^{*2}\,\xi^*_z = 0\,. \tag{26}$$

With the help of Eqs. (23), (24), (26) we obtain the differential equation satisfied by ξ^*_z:

$$\omega^{*2}\,\nu^{*2}_1\,\beta^*\,D^{*3}\,\xi^*_z + 2[S^2(\omega^{*2} - k^{*2}\,S^2) + \omega^{*2}\,\nu^{*2}_1\,k^{*2} - \omega^{*2}\,\nu^{*2}_1\,\beta^{*2}/2]D^{*2}\,\xi^*_z$$
$$- 2[(g^{*2} + \beta^*\,S^2)(\omega^{*2} - k^{*2}\,S^2) + \omega^{*2}\,\nu^{*2}_1\,\beta^*\,(k^{*2} + \beta^*/4 - \beta^{*2}/8)]D^*\,\xi^*_z$$
$$+ [\omega^{*2}\,\nu^{*2}_1\,\beta^*\,k^{*2}\,(\beta^*/4 - \tfrac{1}{2}) + (\omega^{*2} - k^{*2}\,S^2)^2]\,\xi^*_z = 0\,. \tag{27}$$

Expanding $\exp(\beta z)$ and neglecting the terms of the order of $\nu^{*2}_0\,\beta^*$, the above differential equations reduces to

$$[\omega^{*2}\,k^{*2}\,\nu^{*2}_0 + S^2(\omega^{*2} - k^{*2}\,S^2)]D^{*2}\,\xi^*_z - (g^{*2} + \beta^*\,S^2)(\omega^{*2} - k^{*2}\,S^2)D^*\,\xi^*_z + (\omega^{*2} - k^{*2}\,S^2)\xi^*_z = 0\,, \tag{28}$$

where

$$\nu^*_0 = T/(4\,m\,\omega_c)\,. $$

The solution of the above differential equation appropriate to the boundary condition at $z^* = 0$ i.e. $\xi^*_z = 0$, can be written as

$$\xi^*_z = C_1[\exp(m_1\,z^*) - \exp(m_2\,z^*)]\,, \tag{29}$$

where $m_{1,2}$ are the roots of the equation

$$[\omega^{*2}\,k^{*2}\,\nu^{*2}_0 + S^2(\omega^{*2} - k^{*2}\,S^2)]\,m^2 - (g^{*2} + \beta^*\,S^2) \quad (\omega^{*2} - k^{*2}\,S^2)m + (\omega^{*2} - k^{*2}\,S^2)^2 = 0\,. \tag{30}$$

The second boundary condition requires that $\xi^*_z = 0$ at $z^* = 1$ which amounts to

$$m_1 - m_2 = 2\,i\,l\,\pi\,, \tag{31}$$

where l is an integer. Equation (31) with the help of Eq. (30) leads to the dispersion relation

$$4(k^{*2}\,\nu^{*2}_0 + S^2)\,\omega^{*6} - \omega^{*4}[6\,k^{*2}\,S^4 + 2\,k^{*4}\,S^2\,\nu^{*2}_0 + (g^{*2} + \beta^*\,S^2)^2$$
$$+ a^2(S^2 + k^{*2} + \nu^{*2}_0)^2] + [2\,k^{*2}\,S^2(g^{*2} + \beta^*\,S^2)^2 + 3\,k^{*4}\,S^6 + k^{*6}\,S^4\,\nu^{*2}_0$$
$$+ 2\,k^{*2}\,S^4\,a^2(S^2 + k^{*2}\,\nu^{*2}_0)] - [k^{*4}\,S^4(g^{*2} + \beta^*\,S^2)^2 + 4\,k^{*6}\,S^8 + a^2\,k^{*4}\,S^8] = 0\,, \tag{32}$$

where $\qquad\qquad\qquad\qquad a^2 = 4\,\pi^2\,l^2\,.$

We write Eq. (32) in the form which is suitable for discussion: $\qquad \omega^{*6} - a\,\omega^{*4} + b\,\omega^{*2} - c = 0\,,$ $\qquad$ (33)

where a, b, c respectively are the coefficients of ω^{*4}, ω^{*2} and a constant term, divided by the coefficient of ω^{*6}. It is easy to see that Eq. (33) has all the roots positive because $c > 0$ and $a\,b - c > 0$. Also, if β^* changes to $-\beta^*$ or g^{*2} changes to $-g^{*2}$, i. e. to unstable stratification, we find that the three roots of (33) still remain positive. Hence we conclude that the system is stable for both stable and unstable stratification for the disturbances parallel to the ambient magnetic field.

Transverse Oscillations:

We put $k^*_z = 0$ and $k^*_y = k^*$, then Eqs. (21 − 24) reduce to

$$(\omega^{*2} + \nu^*\,\beta^*\,k^*\,\omega^*)\,\xi^*_x = 0\,,\qquad\qquad (34)$$

$$[\omega^{*2} - (1+S^2)\,k^{*2} + \tfrac{1}{2}\,\nu^*\,\omega^*\,\beta^*\,k^*]\,\xi^*_y + i\,[\,(1+S^2)\,k^*\,D^* - k^*\,g^{*2} - \omega^*\,\nu^*\,D^{*2} + \nu^*\,\omega^*\,\beta^*\,D^*/2]\,\xi^*_z = 0\,,\qquad (35)$$

$$[i\,(g^{*2} - (1+S^2)\,\beta^*)\,k^* + (1+S^2)\,i\,k^*\,D^* + i\,\omega^*\,\nu^*\,(D^{*2} - k^{*2}) - \tfrac{1}{2}\,i\,\omega^*\,\nu^*\,\beta^*\,D^*]\,\xi^*_y$$
$$+ [\omega^{*2} + (1+S^2)\,D^{*2} - (1+S^2)\,\beta^*\,D^* + \tfrac{1}{2}\,\nu^*\,\omega^*\,\beta^*\,k^*]\,\xi^*_z = 0\,.\qquad (36)$$

Eliminating ξ^*_y from Eqs. (35) and (36), we obtain the differential equation for ξ^*_z :

$$[\omega^{*2}(1+S^2) - \nu^*\,\omega^*\,k^*\,g^{*2}]\,D^{*2}\,\xi^*_z - [\beta^*(1+S^2)\{\omega^{*2} - k^{*2}(1+S^2)\} + k^*\,g^{*2}\,\{k^*(1+S^2) - \nu^*\,\omega^*\,\beta^*/2\}]\,D^*\,\xi^*_z$$
$$+ [\omega^{*4} + \nu^*\,k^*\,\beta^*\,\omega^{*3} - k^{*2}(1+S^2)\,\omega^{*2} - k^{*3}(1+S^2)\,\nu^*\,\omega^*\,\beta^*/2]\,\xi^*_z = 0\,.\qquad (37)$$

In deriving Eq. (37), the restriction div $\boldsymbol{\xi}^* = 0$, has not been imposed and ν^{*2}, $\nu^*\,D\nu^*$, $\nu^*\,\beta^{*2}$, $\nu^{*2}\,\beta$ etc. have been neglected.

For the system under consideration, to be stable, we assume that $g^{*2} < 1$ [Yu, 1965]. Expanding $\exp(\beta^*\,z^*)$ and retaining only terms of the order of $\nu^*\,\beta^*$, we are left with a differential equation with constant coefficients if we neglect $\nu^*\,\beta^*\,g^{*2}$. The omission of the term $\nu^*\,\beta^*\,g^{*2}$ may be a little more stringent but the variation of gyroviscosity has been taken into account through the differential operator.

Applying the usual foregoing boundary conditions as in case I, we obtain the dispersion relation:

$$4\,\omega^{*6}(1+S^2) + 4\,\nu^*\,k^*\,\omega^{*5}\,[\beta^*(1+S^2) - g^{*2}] + (1+S^2)\,\omega^{*4}[\beta^{*2} - 4\,k^{*2} + 4\,\pi^2\,l^2] + \nu^*\,k^*\,\omega^{*3}$$
$$\times\ (1+S^2)\,[4\,k^{*2}\,g^{*2} - 2\,k^{*2}(1-S^2)\,\beta^* - 8\,\pi^2\,l^2\,g^{*2}] + 2\,\omega^{*2}(1+S^2)^2\,k^{*2}\,\beta^*\,[g^{*2} - \beta^*(1+S^2)]$$
$$+ \nu^*\,\omega^*\,k^{*3}\,g^{*2}\,\beta^*(1+S^2)\,[(1+S^2) - g^{*2}] + k^{*4}(1+S^2)^2\,[\beta^{*2}(1+S^2)^2 + g^{*4} - 2\,\beta^*\,g^{*2}(1+S^2)] = 0\,,\quad (38)$$

where l is any integer.

We discuss the dispersion relation (38) in two limiting situations a) $k \to \infty$, b) $k^* \to 0$.

C a s e I   Short waves: $k \to \infty$.

The dispersion relation (38) in this limit yields the following modes when $\nu^* = 0$:

$$\omega^{*2} = k^{*2}(1+S^2)\,,\qquad\qquad (39)$$

$$\omega^{*4} = -\,\frac{k^{*4}}{4}\,\beta^*\,g^{*2}(1+S^2)\,[2 - \beta^*(1+S^2)]/(k^{*2} - \pi^2\,l^2)\,.\qquad (40)$$

Equations (39) and (40) respectively lead to stable and unstable situations. The correction to the modes [Eq. (39)] when $\nu^* \neq 0$, is given by

$$\delta\omega^* = -\,\frac{\nu^*(1+S^2)\,k^{*3}[2\,g^{*2} - (1+S^2)\,\beta^*/2] + 2\,\nu^*\,k^*\,\omega^{*2}[\beta^*(1+S^2) - g^{*2}]}{6(1+S^2)\,\omega^{*2} + 4\,\nu^*\,k^*\,[\beta^*(1+S^2) - g^{*2}]\,\omega^* - 2\,k^{*2}(1+S^2)^2}\,.\qquad (41)$$

From Eq. (41), it can be seen that the gyro-viscosity damps the oscillations. The correction to the unstable modes given by Eq. (40), when $\nu^* \neq 0$, is

$$\delta\omega^* = \frac{\nu^* k^* \omega^* [2 k^{*2} g^{*2} - k^{*2}(1+S^2)\beta^* - 4\pi^2 l^2 g^{*2}]}{8 \omega^*(1+S^2)(k^{*2}-\pi^2 l^2) + 3\nu^* k^*(1+S^2)[2 k^{*2} g^{*2} - \beta^* k^{*2}(1+S^2) - 4\pi^2 l^2 g^{*2}]} . \tag{42}$$

Equation (42) gives negative $\delta\omega^*$ for large k^* and so we conclude that it has stabilizing influence over the unstable modes given by Equation (40).
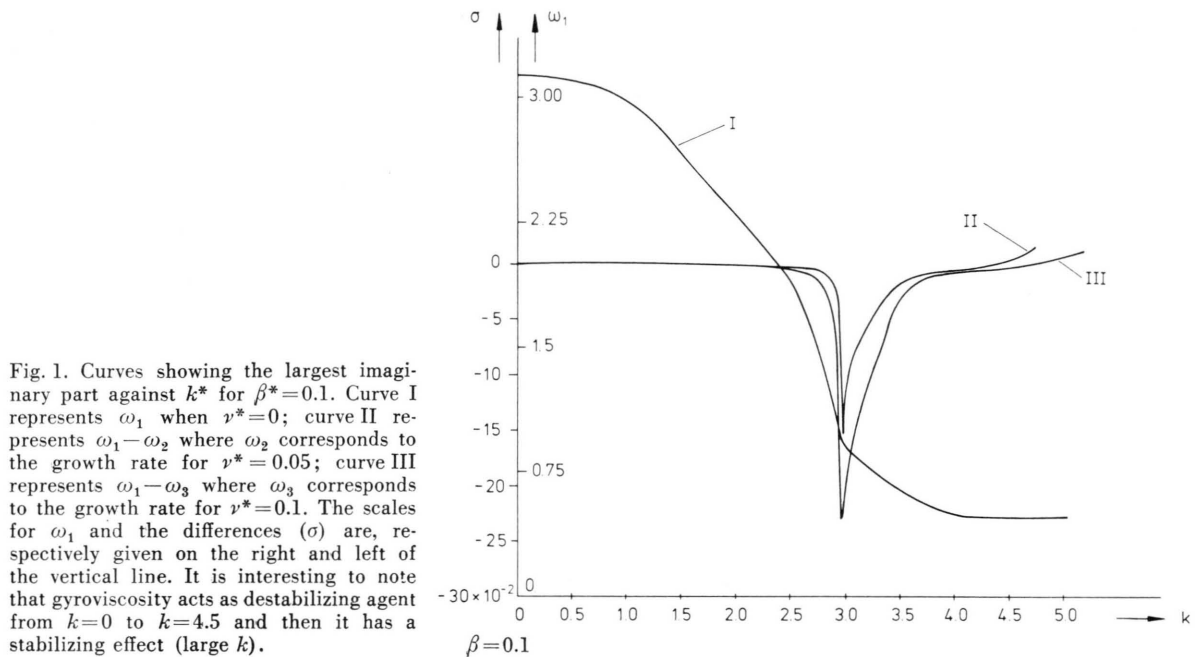

Fig. 1. Curves showing the largest imaginary part against k^* for $\beta^* = 0.1$. Curve I represents ω_1 when $\nu^* = 0$; curve II represents $\omega_1 - \omega_2$ where ω_2 corresponds to the growth rate for $\nu^* = 0.05$; curve III represents $\omega_1 - \omega_3$ where ω_3 corresponds to the growth rate for $\nu^* = 0.1$. The scales for ω_1 and the differences (σ) are, respectively given on the right and left of the vertical line. It is interesting to note that gyroviscosity acts as destabilizing agent from $k=0$ to $k=4.5$ and then it has a stabilizing effect (large k).
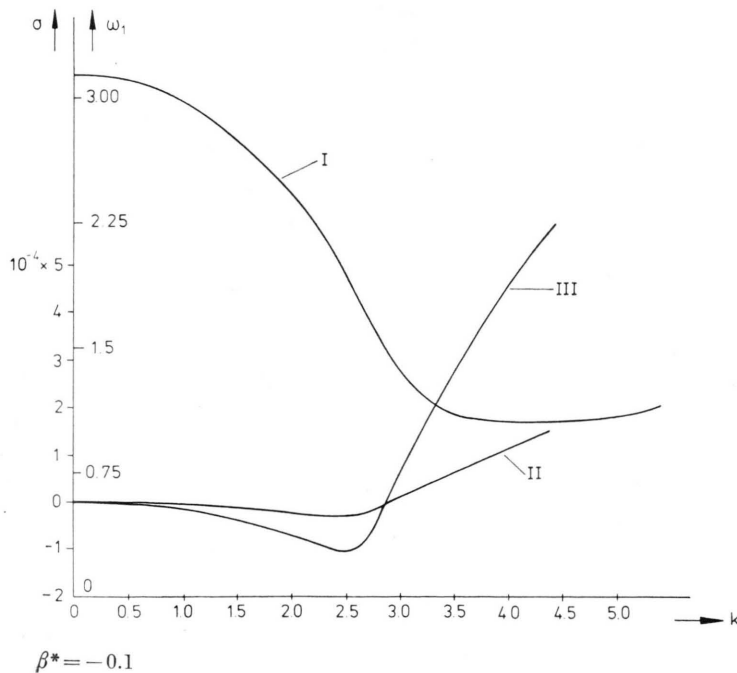

Fig. 2. Curves showing the largest imaginary part against k^* for $\beta^* = -0.1$. Curve I represents ω_1 when $\nu^* = 0$; curve II represents $\omega_1 - \omega_2$ where ω_2 corresponds to the growth rate for $\nu^* = 0.05$; curve III represents $\omega_1 - \omega_3$ where ω_3 corresponds to the growth rate for $\nu^* = 0.1$. The scales for ω_1 and the differences (σ) are, respectively given on the right and left of the vertical line. It is interesting to note that gyroviscosity acts as destabilizing agent from $k=0$ to $k=2.85$ and then it has a stabilizing effect (large k).

Case II   Long waves: $k \to 0$.

The dispersion relation (38) in this case gives (neglecting k^4 and k^3):

$$\omega^{*2} = \tfrac{1}{8}\left[(1+S^2)(4\,k^{*2}-\beta^{*2}-4\,\pi^2\,l^2) \pm \left\{(1+S^2)^2(\beta^{*2}-4\,k^{*2}+4\,\pi^2\,l^2)^2 \right.\right.$$
$$\left.\left. -32\,\beta^*\,k^{*2}(1+S^2)[g^{*2}-\beta^*(1+S^2)]\right\}^{1/2}\right]. \tag{43}$$

From (43), we see that the long wave perturbations are unstable. The shift in frequency for $\nu^* \neq 0$ is given by

$$\delta\omega^* = -\nu^*[2\,k^*\,\omega^{*3}\left(\beta^*(1+S^2)-g^{*2}\right)-4\,\omega^*(1+S^2)\,k^*\,\pi^2\,l^2g^{*2}]/[8\,\omega^{*3}(1+S^2)$$
$$+6\,\nu^*\,k^*\,\omega^{*2}\left(\beta^*(1+S^2)-g^{*2}\right)+\omega^*(1+S^2)^2(\beta^{*2}-4\,k^{*2}+4\,\pi^2\,l^2)-4\,\pi^2\,l^2\,g^{*2}\,\nu^*(1+S^2)]. \tag{44}$$

As ω^* is purely imaginery for small k^*, $\delta\omega^*$ is negative imaginary. We conclude that long-wave disturbances are destabilized by gyro-viscosity.

To exhibit some important points, we obtain the roots of the dispersion relation (38) with the help of the computer for the following values of the physical parameters:

$$\nu^* = 0,\, 0.05,\, 0.1, \qquad S^2 = 1,\quad g^2 = 0.1, \qquad \beta = 0.1,\, -0.1,\quad l = 1.$$

The largest imaginary part has been plotted against k in Figures 1 and 2.

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